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Oppolzer's and Ginzel's Corrections to Hansen.

By J. K. Fotheringham.

Oppolzer, in his *Syzygientafeln* (1881), p. 15, proposes certain corrections to Hansen's values for the mean motions and accelerations of the Moon. As these corrections have been applied in the calculation of Oppolzer's *Canon der Finsternisse* (1887), it is important that they should be correctly interpreted by those who have occasion to use the *Canon der Finsternisse.*

Oppolzer gives his corrections in the form—

$$\begin{array}{rllll} \Delta T = & +0{\cdot}0006\,s & +0{\cdot}00009\,s^2 & +0{\cdot}000 & 00009\,s^3 \\ \Delta(g+\omega) = & -0^\circ 019\,s & -0^\circ 0004\,s^2 & -0^\circ 000 & 0004\,s^3 \\ \Delta g = & & +0^\circ 003\,s^2 & +0^\circ 000 & 003\,s^3 \end{array}$$

where s is the interval in centuries since 1800·0,
T is the time of mean syzygy expressed in decimals of a day,
g is the mean anomaly of the Moon,
ω is the longitude of lunar perigee measured from the ascending node.

As Oppolzer's tables exist for the purpose of computing the elements of a syzygy, not of constructing an ephemeris, his corrections naturally apply to the moment of mean syzygy, not to a fixed moment of time; further, since his tables express g in centesimal degrees, and $g+\omega$ in sexagesimal degrees, it seems reasonable to suppose that the corrections are expressed in the same form. In order to make sure that these principles of interpretation are correct, I have computed the corrections for −101 and −462, and find that they only agree with the corrections actually applied if interpreted as described above. Unfortunately, Oppolzer gives no warning as to the interpretation of his corrections, and they have in consequence been frequently misunderstood. The misunderstanding is rendered the easier by the use of the symbol ° for centesimal degrees. I have not found any other passage in Oppolzer where that symbol is used for any but sexagesimal degrees.

Professor Ginzel, in his *Astronomische Untersuchungen über Finsternisse* in *Sitzungsberichte der kaiserlichen Akademie der Wissenschaften math. naturw. Classe*, lxxxix. (2), (1884), uses Oppolzer's *Syzygientafeln* as the basis of his corrections, and, while realising that the corrections are to be applied to the moment of mean syzygy, interprets Δg as if it were expressed in sexagesimal degrees. The corrections thus obtained are, however, tested by the eclipses used and made the basis of further corrections, which are not affected by the misunderstanding of Oppolzer's Δg. Dr. Schram, in his *Reductionstafeln* (*Denkschriften der k. Akademie der W. math. naturw. cl.*, lvi.) (1889), in reducing Professor Ginzel's

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corrections so as to be more directly applicable to Oppolzer's *Canon*, interprets Oppolzer's corrections rightly; and, as Professor Ginzel's *Spezieller Kanon* (1899) is based on Dr. Schram's *Reductionstafeln*, it is not affected by Professor Ginzel's error in the interpretation of Oppolzer.

When Oppolzer's own collaborator, Professor Ginzel, misunderstood him, it is not unnatural that he should have been misunderstood to an even greater extent by others. Professor Newcomb, in *Astronomische Nachrichten*, 154 (1901), pp. 197–202, discusses Oppolzer's and Ginzel's corrections and appears to misunderstand both. Professor Ginzel expresses his corrections in a form similar to Oppolzer's. Those corresponding to Oppolzer's corrections are expressed as follows:—

$$\Delta T = -0{\cdot}000192\, s + 0{\cdot}0000247\left(1 + \frac{s}{1000}\right) s^2$$

$$\Delta P = \Delta Q = -0^\circ{\cdot}00769\, s \quad + 0^\circ{\cdot}000049\left(1 + \frac{s}{1000}\right) s^2$$

$$\Delta I = +0{\cdot}0240\, s \quad + 0{\cdot}00052\left(1 + \frac{s}{1000}\right) s^2$$

T and s are clearly intended to have the same meaning here as in Oppolzer, though Professor Ginzel (*Spezieller Kanon*, p. 5) defines s as the number of years since 1800·0; ΔP is the same as Oppolzer's $\Delta\ (g + \omega)$ and ΔI is the same as Oppolzer's Δg and is expressed in centesimal degrees. These corrections were intended by Professor Ginzel to be substituted for Oppolzer's. Professor Newcomb, in the article to which I have referred, reduces both Oppolzer's and Ginzel's corrections to the form of corrections of the longitude of the Moon and of its ascending node and perigee. He reduces ΔT correctly into a correction of the moon's longitude (ΔL) for a given time by multiplying it by the factor $-43900''$. He then deducts from this correction Oppolzer's $\Delta(g + \omega)$ and Professor Ginzel's ΔP respectively and calls the remainder $\Delta\Omega$. This would be legitimate if $\Delta(g + \omega)$ and ΔP referred to a fixed moment of time; but since they refer to the moment of mean syzygy, it is necessary to deduct from $\Delta(g + \omega)$ and ΔP respectively the motion of $(g + \omega)$ between Hansen's and the corrected moment of mean syzygy before comparing $\Delta(g + \omega)$ with ΔL. Professor Newcomb next interprets Oppolzer's Δg and Professor Ginzel's ΔI as referring to sexagesimal degrees, deducts each from ΔL, and calls the remainders $\Delta\pi$. Then he attacks the corrections of perigee and node thus computed, asserting that the corrections of the centennial motions are incompatible with the observations of the last 200 years, and that the corrections of the accelerations are incompatible with gravitational theory. This result is a little surprising in the case of Professor Ginzel's secular accelerations of the node and perigee, which profess to be taken direct from Delaunay.

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Professor's Newcomb's reduction of Oppolzer's and Ginzel's corrections is as follows:—

	Oppolzer.		Ginzel.	
$\Delta L =$	$-26''{\cdot}3\,s$	$-\ 3''{\cdot}95\,s^2$	$+\ 8''{\cdot}4\,s$	$-1''{\cdot}08\,s^2$
$\Delta\Omega =$	$+42\ \ s$	$-\ 2{\cdot}5\ \ s^2$	$+36{\cdot}1\,s$	$-1{\cdot}26\,s^2$
$\Delta\pi =$	$-26\ \ s$	$-14{\cdot}7\ \ s^2$	$-78{\cdot}0\,s$	$-2{\cdot}95\,s^2$

Allowing for the motion of the Moon and its perigee and node between Hansen's and the corrected moment of mean syzygy, and converting Δg and Δl from the centesimal to the sexagesimal system, I reduce as follows:—

	Oppolzer.		Ginzel.	
$\Delta L =$	$-26''{\cdot}3\,s$	$-3''{\cdot}95\,s^2$	$+\ 8''{\cdot}4\,s$	$-1''{\cdot}08\,s^2$
$\Delta\Omega =$	$+71\ \ s$	$+1{\cdot}8\ \ s^2$	$+27{\cdot}0\,s$	$-0{\cdot}08\,s^2$
$\Delta\pi =$	$+\ 2\ \ s$	$-9{\cdot}4\ \ s^2$	$-78{\cdot}4\,s$	$-1{\cdot}61\,s^2$

From this it will appear that Oppolzer's correction to the centennial motion of perigee falls within the limit (10″ or a little more) of error which Professor Newcomb allows, though his other corrections are still open to objection. Professor Ginzel's corrections of the centennial motions of node and perigee are, as Professor Newcomb says, incompatible with modern observations, but it will be observed that his corrections of the accelerations of node and perigee are in exact agreement with gravitational theory, while his correction of the centennial motion of the argument of latitude $(L-\Omega)$ is at least superior to the correction adopted in the *Nautical Almanac.* It would appear, then, that Professor Newcomb has overstated the case against Oppolzer and Ginzel, but that he is right in regarding some of their corrections as impossible.

In order to render the effect of these corrections the clearer, I append the lunar and solar formulæ which result from them.

Oppolzer.

$$
\begin{aligned}
g &= 110^\circ\ 19'\ 33''{\cdot}64 + 171791\ 5808''{\cdot}93\,T + 56''{\cdot}623\,T^2 + 0''{\cdot}056639\,T^3 \\
\omega &= 192\ \ \ 7\ \ 27{\cdot}41 + \ \ 2161\ 1442{\cdot}28\,T - 56{\cdot}417\,T^2 - 0{\cdot}056432\,T^3 \\
\Omega &= \ \ 33\ \ 16\ \ 25{\cdot}65 - \ \ \ 696\ 2857{\cdot}94\,T + \ 9{\cdot}522\,T^2 + 0{\cdot}008403\,T^3 \\
L' &= 279\ \ 54\ \ 49{\cdot}21 + \ 12960\ 2766{\cdot}73\,T + \ 1{\cdot}1073\,T^2 \\
\pi' &= 279\ \ 30\ \ 17{\cdot}35 + \qquad\ \ 6168{\cdot}13\,T + \ 1{\cdot}823\,T^2
\end{aligned}
$$

Ginzel.

$$
\begin{aligned}
g &= 110^\circ\ 19'\ 33''{\cdot}64 + 171791\ 5923''{\cdot}93\,T + 51''{\cdot}657\,T^2 + 0''{\cdot}051673\,T^3 \\
\omega &= 192\ \ \ 7\ \ 27{\cdot}41 + \ \ 2161\ 1405{\cdot}75\,T - 46{\cdot}723\,T^2 - 0{\cdot}046738\,T^3 \\
\Omega &= \ \ 33\ \ 16\ \ 25{\cdot}65 - \ \ \ 696\ 2901{\cdot}65\,T + \ 7{\cdot}660\,T^2 + 0{\cdot}006541\,T^3 \\
L' &= 279\ \ 54\ \ 49{\cdot}21 + \ 12960\ 2766{\cdot}73\,T + \ 1{\cdot}1073\,T^2 \\
\pi' &= 279\ \ 30\ \ 17{\cdot}35 + \qquad\ \ 6168{\cdot}13\,T + \ 1{\cdot}823\,T^2
\end{aligned}
$$

As Oppolzer treats as negligible the change in the mean anomaly of the Sun between Hansen's mean syzygy and his own, the values actually represented by his tables imply a corresponding correction to the solar perigee. This would make

$$\pi' = 279^\circ\ 30'\ 17''\!\cdot\!35 + 6170''\!\cdot\!26\,T + 2''\!\cdot\!142\,T^2 + 0''\!\cdot\!000319\,T^3$$

No such correction is necessary in the case of Professor Ginzel's values, since he has introduced corrections which make the neccssary allowance for the change in the Sun's mean anomaly.

Mr. Nevill, in *Monthly Notices*, lxvii. p. 14, explains that in his recent discussion of ancient eclipses he worked with Oppolzer's tables, applying certain corrections, and he proceeds to give corrections to Hansen's tables, to which the elements employed by Oppolzer are stated to correspond. His reductions are obtained in the same way as those given by Professor Newcomb, but they are expressed as corrections of the elements in Hansen's tables, not of the corrected elements, mainly from the *Darlegung*, used by Oppolzer. To these Mr. Nevill applies further corrections in order to bring the centennial motions into accord with modern observations, and the secular accelerations into accord with theory.

He thus obtains the following corrections to be applied to Oppolzer:—

$$\begin{aligned}
\text{(Mean longitude) } \Delta\zeta &= +0''\!\cdot\!00 - 3''\!\cdot\!66\,T - 2''\!\cdot\!496\,T^2\\
\text{(Longitude of perigee) } \Delta A &= +0\cdot00 + 37\cdot14\,T + 13\cdot128\,T^2\\
\text{(Longitude of node) } \Delta B &= +0\cdot00 - 48\cdot06\,T + 2\cdot672\,T^2
\end{aligned}$$

Accepting Mr. Nevill's values for the results of modern observations and theory, I find that these corrections should have been—

$$\begin{aligned}
\Delta\zeta &= +0''\!\cdot\!00 - 3''\!\cdot\!66\,T - 2''\!\cdot\!496\,T^2\\
\Delta A &= +0\cdot00 + 8\cdot92\,T + 7\cdot817\,T^2\\
\Delta B &= +0\cdot00 - 76\cdot67\,T - 1\cdot617\,T^2
\end{aligned}$$

The effect of Mr. Nevill's misunderstanding of Oppolzer is that he has unconsciously applied a further correction:—

$$\begin{aligned}
\Delta A &= +28''\!\cdot\!22\,T + 5''\!\cdot\!311\,T^2\\
\Delta B &= +28\cdot61\,T + 4\cdot289\,T^2
\end{aligned}$$

With the corrections obtained in this manner Mr. Nevill calculates, with the aid of Oppolzer's *Canon*, the elements of a number of eclipses, and gives the result as that which accords with modern theory. He also reduces three other sets of lunar and solar values, including Mr. Cowell's, to the form of corrections to be applied to Oppolzer, interpreting Oppolzer as before, and therefore making the same error in the reduction. It is no wonder that Mr. Cowell

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should complain that the results which Mr. Nevill deduces from his values differ widely from those which he deduces himself. As Mr. Nevill's error is a large one, it seems to follow that his calculations must be set aside, and if different theories of lunar motion are to be tested by ancient eclipses, the work will have to be done afresh.

It does not appear to me that the errors in Professor Ginzel's centennial motions are a sufficient reason for setting aside his elaborate computations of ancient eclipses. If a set of values, consistent with modern observation and with theory, would give the same results for ancient times, Professor Ginzel's computations would be evidence in favour of such a set of values. The most important elements in eclipse calculations are elongation (D) and the argument of latitude (F). Professor Ginzel's mean motion of D exceeds Mr. Cowell's (based on observation) by $38''$, and his mean motion of F exceeds Mr. Cowell's by $20''$. If we substitute Mr. Cowell's mean motions for Professor Ginzel's, we may, by making corresponding reductions in Professor Ginzel's secular terms, still obtain approximately the same results for ancient eclipses. Taking -300 as a mean date for the ancient eclipses falling under discussion, we must reduce Professor Ginzel's coefficients of the secular terms of D and F by $\frac{38''}{21} = 1''{\cdot}8$ and $\frac{20''}{21} = 0''{\cdot}9$ respectively. This gives $+9''{\cdot}7$ for the secular coefficient of D and $+4''{\cdot}0$ for the secular coefficient of F. The resultant secular coefficient of D − F ($5''{\cdot}7$) does not differ greatly from the theoretical coefficient ($6''{\cdot}55$). It is not likely that any one would propose a secular acceleration of the Sun to account for so small a difference, for it ought not to be difficult to make small re-arrangements of the different values so as to bring this coefficient within the range permitted by modern theory without seriously affecting the result in ancient times. It may therefore be said that Professor Ginzel's elements of ancient eclipses roughly represent modern observation and theory for the motion of the node and mean motion of the Moon, with a value of $9''{\cdot}7$ for secular acceleration; and if his tables should be found to do justice to the historical data, there would be no case for a secular acceleration of the Sun. Mr. Cowell's values have, so far as they have been tried, yielded better results, but it would be interesting to have the two sets of values compared more thoroughly than has yet been done.

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Historical Data for the Secular Acceleration of the Moon.

By J. K. Fotheringham.

Father Kugler, in his recent volume on Babylonian Astronomy,* compares the new moon longitudes given in the lunar tables of Bania and Marduk-tabik-ziru constructed in – 102 with the longitudes computed by means of Oppolzer's tables, and concludes that Oppolzer's new moon longitudes are on an average 1° 2′ too high. The accuracy of the constants employed by Bania and Marduk-tabik-ziru is so great, and the observations on which these tables were based were presumably so nearly contemporary, that Father Kugler infers that at least the greater part of the error must lie in the modern, and not in the Babylonian tables. Now, it is incredible that any error approaching this can exist in our modern tables, whose substantial accuracy is confirmed by numerous ancient eclipses, and I have therefore thought it worth while to make a fuller analysis of the tables of Bania and Marduk-tabik-ziru, especially as these, if accurate, ought to furnish us with corrections not only to the Moon's longitude, but to her perigee and node, as also to the longitude and perigee of the Sun.

The most important part of these tables will be found in Father Kugler's *Babylonische Mondrechnung* (1900), pp. 12, 13, where they are accompanied by a very full discussion. The tables are dated at Sippara, – 102 December 22, and extend from – 103 March 22 to – 100 April 17. The lunar constants are, as Father Kugler has shown, identical with those used by Hipparchus, but the Babylonian solar constants appear to be less accurate than those of the Greek astronomer. Father Kugler has computed the sidereal, synodical, anomalistic, and nodical months, and also the sidereal and anomalistic years, by means of these tables, and compares the result with their values in 1800·0, but he has not computed any places except the longitudes of new moon. I have preferred to deduce from these tables the centennial motions and mean places at the epoch – 101 December 20·0, G.M.T., exactly 1900 years before the epoch of Hansen's tables, and to compute the secular accelerations which result from the comparison of these with the centennial motions and mean places deduced by Mr. Cowell from modern observations, and published in *Monthly Notices*, lxv. p. 863. As the Babylonian mean motions are sidereal, I have subjected Mr. Cowell's centennial motions to a correction for precession, and in computing the Babylonian places I have provisionally accepted Father Kugler's value for the longitude of the Babylonian first point of Aries. This, reduced to the epoch

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* *Sternkunde und Sterndienst in Babel*, i. Buch, *Babylonische Planetenkunde* (Münster, 1907), pp. 172, 173.